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Minimum Aberration Designs Are Not Maximally Unconfounded

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Abstract

This article gives two examples of comparisons of minimum aberration 2^(k-p) designs with alternative designs to show that minimum aberration designs are not maximally unconfounded, in terms of the number of factor interactions that can be estimated and uniquely attributed to the respective effects. Given the traditional definition of design resolution, which pertains to the order and number of effects that can be estimated and unambiguously attributed, the minimum aberration criterion may lead to the selection of designs with fewer such effects than what can be achieved by directly searching for designs that maximize the number of unconfounded effects. The criterion of maximum unconfounding represents a more natural and useful extension of the concept of design resolution than does the criterion of minimum aberration.

Key words: Minimum aberration, Fractional factorial designs, Resolution Confounding

Minimum Aberration Designs Are Not Maximally Unconfounded

Fractional factorial designs with two-level factors, or 2^(k-p) designs, are widely used in industrial experimentation, because they allow the researcher to screen a large number of factors, and their lower-order interactions, in a small number of experimental runs. Box, Hunter, and Hunter (1978) published a table of such designs (Table 12.15, p. 410) which have since come to be regarded as standard designs for 2^(k-p) fractional factorial experiments. The legacy of the Box, Hunter, and Hunter (1978) designs is seen in their inclusion in comprehensive textbooks on experimental design (e.g., Box & Draper, 1987, Gunst, Mason, & Hess, 1989, Montgomery, 1991), and in their use as criteria for demonstrating the adequacy of experimental design software (e.g., procedure FACTEX in SAS, 1995). However, the designs listed by Box, Hunter, and Hunter (1978) are not necessarily the "best" designs, when one is interested in estimating the largest number of unconfounded interactions of the highest order, given the resolution of the design.

Minimum Aberration Designs

Fries and Hunter (1984) showed how the Box, Hunter, and Hunter (1978) designs could be produced by minimizing the aberration of the design. Fries and Hunter (1984) proposed the concept of design aberration as a natural extension of the concept of design resolution. An experimental design is of resolution R if all effects containing s or fewer factors are unconfounded with any effects containing fewer than R - s factors. Alternatively, resolution can be defined as the length of the shortest word in the defining relation for the respective design (e.g., a Resolution IV design has at least one word of length 4 in its defining relation, see Box & Draper, 1987, or Mason, Gunst, & Hess, 1989, for descriptions of the procedure for determining the words in the identity relation for a design).

Fries and Hunter (1984) defined the minimum aberration design as the design of maximum resolution R "which minimizes the number of words in the defining relation that are of minimum length" (p. 602). Fries and Hunter (1984) further state that, "Given that resolution is maximized and equal to *Rmax*, minimizing aberration ensures that a design has the minimum number of words of length *Rmax*, which, in turn, means that the smallest number of main effects will be confounded with interactions of order *Rmax* - 1, the smallest number of two-factor interactions will be confounded with interactions of order *Rmax* - 2, and so forth. Hence, the concept of aberration is a natural extension of resolution." (p. 602).

Maximally Unconfounded Designs

The appeal of the minimum aberration criterion is in the suggestion that, extending the concept of design resolution, it will produce designs with the maximum number of effects of the "crucial order" s (e.g., two-factor interactions in the case of Resolution IV designs), given the resolution, which are unconfounded with any effects containing R - s factors (e.g., other two-factor interactions in the case of Resolution IV designs). However, this is not the case. To illustrate, Table 1 lists the generators, the identity relations, and the aliasing for the two-factor interactions for the Box, Hunter, and Hunter (1978) Resolution IV, 2 ^(9-4) design (the Standard Design), and an alternative design: the maximally unconfounded design. The latter design was derived by searching all non-isomorphic sets of generators, choosing a set that produces the maximum number of unconfounded two-factor interactions (with other two-factor interactions).

Table 1: Generators, Identity Relations, and Aliasing for Two 2^(9-4) Designs

Minimum Aberration Design (Box, Hunter, and Hunter, 1978):

Generators:

```
6 = 2345
7 = 1345
8 = 1245
9 = 1235
```

Identity Relation:

```
I = 23456=13457=12458=12359              the generators
  = 1267=1368=1469=2378=2479=3489        the 2-way products of generatorS
  = 45678=35679=25689=15789              the 3-way products of generatorS
  = 12346789                             the 4-way product of generators
```

Aliasing of two-factor interactions:

```
12=67
13=68
```

```
14=69
15
16=27=38=49
17=26
18=36
19=46
23=78
24=79
25
28=37
29=47
34=89
35
39=48
45
56
57
58
59
```

Maximally Unconfounded Design:

Generators:

```
6 = 1234
7 = 1235
8 = 1245
9 = 345
```

Identity Relation:

I = 12346=12357=12458=3459	the generators
= 4567=3568=12569=3478=12479=12389	the 2-way products of generators
= 12678=3679=4689=5789	the 3-way products of generators
= 123456789	the 4-way product of generators

Aliasing of two-factor interactions:

```
12
13
14
15
16
17
18
19
23
```

24
25
26
27
28
29
34=59=78
35=49=68
36=58=79
37=48=69
38=47=56
39=45=67
46=57=89

In Table 1, the word length patterns for the identity relations show that the standard design has 6 words of length 4 and that the maximally unconfounded design has 7 words of length 4, thus, according to the minimum aberration criterion, the standard design should be the better design. However, in the standard design, only 8 of the two-factor interaction effects are unconfounded with any other two-factor interaction, while in the maximally unconfounded design, 15 of the two-factor interactions are unconfounded with any other two-factor interaction. It should be noted that the standard minimum aberration design produces 18 pairs of confounded interactions, while the second design produces 21 pairs of confounded interactions.

Usefulness of the Minimum Aberration Criterion

The question thus arises, which one of these criteria is more useful: minimizing the confounding with any other effects (maximizing the number of unconfounded effects) or minimizing the confounding between pairs of effects. Confounding of effects is the bane of the experimenter. According to Mason, Gunst, and Hess (1989), "Two or more experimental effects are confounded if calculated effects can only be attributed to their combined influence on the response, not to their individual ones. Two or more effects are confounded if the calculation of one effect uses the same (apart from sign) difference or contrast of the response averages as the calculation of other effects." The logic advocating the elimination, whenever possible, of confounding in experimental designs is that if one can unambiguously attribute an effect to a specific interaction, this is preferable to being able to estimate an effect but not to be able to attribute it to a specific interaction.

The concept of design resolution is based on this reasoning, that is, to maximize the number of effects of a certain order that can be estimated and unambiguously attributed to the respective effects. Minimum aberration designs do not produce this desirable quality of maximizing the number of higher-order interactions that can be estimated, and unambiguously attributed to the respective effects. However, the criterion of maximum unconfounding represents a natural extension of the concept of design resolution.

Differences in Minimum Aberration vs. Maximally Unconfounded Designs

Using both criteria discussed above, we evaluated the standard designs with up to 11 factors presented in Table 12.15 of Box, Hunter, & Hunter (1978). The Resolution IV, 2^(9-4) design is the only design which is not maximally unconfounded (see also Table 1). However, for designs with more than 11 factors, very different designs will be selected when using these two criteria. In those cases, the minimum aberration criterion appears to lead to increasingly fewer unconfounded factor-interaction effects, as the number of factors increases. To illustrate, Table 2 lists the generators and the aliasing for the two-factor interactions for the minimum aberration, Resolution IV, 2 ^(15-9) design, and the corresponding maximally unconfounded design.

Table 2: Generators and Aliasing for two 2^(15-9) Designs

Minimum Aberration Design:

Generators:

```
 7 = 1 2 3 4 5 6
 8 = 3 4 5 6
 9 = 2 4 5 6
10 = 2 3 5 6
11 = 1 5 6
12 = 2 3 4 6
13 = 1 4 6
14 = 1 3 6
15 = 1 2 6
```

Aliasing of two-factor interactions:

```
1   2 =  6 15 =  7  8
1   3 =  6 14 =  7  9
1   4 =  6 13 =  7 10
1   5 =  6 11 =  7 12
1   6 =  2 15 =  3 14 = 4 13 = 5 11
1   7 =  2  8 =  3  9 = 4 10 = 5 12
1   8 =  2  7
1   9 =  3  7
1  10 =  4  7
1  11 =  5  6
1  12 =  5  7
1  13 =  4  6
1   4 =  3  6
1   5 =  2  6
```

```
2   3 =  8  9 = 14 15
2   4 =  8 10 = 13 15
2   5 =  8 12 = 11 15
2   9 =  3  8
2  10 =  4  8
2  11 =  5 15
2  12 =  5  8
2  13 =  4 15
2  14 =  3 15
3   4 =  9 10 = 13 14
3   5 =  9 12 = 11 14
3  10 =  4  9
3  11 =  5 14
3  12 =  5  9
3  13 =  4 14
4   5 = 10 12 = 11 13
4  11 =  5 13
4  12 =  5 10
6   7 =  8 15 =  9 14 = 10 13 = 11 12
6   9 =  7 14
6  10 =  7 13
6  12 =  7 11
8  11 = 12 15
8  13 = 10 15
9  11 = 12 14
9  13 = 10 14
10 11 = 12 13
```

Maximally Unconfounded Design:

Generators:

```
7  = 1 2 3 4 5 6 7
8  = 1 2 3 4
9  = 1 2 3 5
10 = 1 2 3 6
11 = 1 2 4 5
12 = 1 2 4 6
13 = 1 2 5 6
14 = 3 4 5
15 = 3 4 6
```

Aliasing of two-factor interactions:

```
1  2
1  3
1  4
1  5
1  6
1  7
1  8
1  9
1 10
1 11
1 12
1 13
1 14
1 15
2  3
2  4
2  5
2  6
2  7
2  8
2  9
2 10
2 11
2 12
2 13
2 14
2 15
3  4 = 5 14 = 6 15 =  7 13 =  9 11 = 10 12
3  5 = 4 14 = 7 12 =  8 11 = 10 13
3  6 = 4 15 = 7 11 =  8 12 =  9 13
3  7 = 4 13 = 5 12 =  6 11 =  9 15 = 10 14
3  8 = 5 11 = 6 12 =  9 14 = 10 15
3  9 = 4 11 = 6 13 =  7 15 =  8 14
3 10 = 4 12 = 5 13 =  7 14 =  8 15
3 11 = 4  9 = 5  8 =  6  7 = 13 15
3 12 = 4 10 = 5  7 =  6  8 = 13 14
3 13 = 4  7 = 5 10 =  6  9 = 11 15 = 12 14
3 14 = 4  5 = 7 10 =  8  9 = 12 13
3 15 = 4  6 = 7  9 =  8 10 = 11 13
4  8 = 5  9 = 6 10 = 11 14 = 12 15
5  6 = 7  8 = 9 10 = 11 12 = 14 15
5 15 = 6 14 = 8 13 =  9 12 = 10 11
```

For the two designs presented in Table 2, the word length patterns for the identity relations (which contain 2^p - 1, or 511 words for each design, and which are not shown due to space limitations) show that the minimum aberration design has 30 words of length 4 and that the maximally unconfounded design has 55 words of length 4. However, in the minimum aberration design, no two-factor interaction effects are unconfounded with any other two-factor interaction, while in the maximally unconfounded design, 27 of the two-factor interactions (i.e., all two-factor interactions involving factor 1 or factor 2) are unconfounded with any other interaction. The minimum aberration design does outperform the maximally unconfounded design in terms of the number of pairs of aliased effects (90 and 145 pairs of aliased interactions, respectively), and in terms of the total number of estimable two-factor interaction effects (43 and 42 estimable effects, respectively).

The comparison of the two 2^(15-9) fractional factorial designs presented in Table 2 reveals the limited sense in which minimum aberration designs could be argued to be minimally confounded. Apparently, the minimum word length criterion leads to the selection of designs with the minimum number of *pairs* of aliased effects; but minimizing the number of *pairs* of aliased effects does not necessarily result in maximizing the number of effects that are unconfounded (with any other effects of the crucial order).

Conclusion

The upshot of this discussion is that minimum aberration fractional factorial designs are not always maximally unconfounded designs, and that therefore the minimum aberration criterion is of limited usefulness for designing fractional factorial experiments. Fries and Hunter (1984) proposed the concept of minimum aberration as a natural extension of the concept of design resolution. This paper shows that the criterion of maximum unconfounding represents the more natural and useful extension of the concept of design resolution; this criterion can be used to select design generators that will allow the experimenter to estimate and unambiguously attribute (and interpret) the effects of the largest number of factor-interactions of a particular order, given the resolution of the design.

Fractional factorial designs were originally developed because of the economy of data collection they provide, especially in programs of research that sequentially inspect the effects of additional factors, iteratively building on previously acquired information. To give a concrete example, suppose an engineer had previously investigated the effects of four factors and their two-factor interactions on some manufacturing process. The effects of two new factors are desired to be investigated. The engineer might well design a Resolution IV, 2^(15-9) experiment, because under the assumption that all three-factor and higher interactions are zero, such an experiment would provide unconfounded estimates of the main effects for all 6 factors, might provide unconfounded estimates of the effects of at least some two-factor interactions, and could be conducted with as little as 64 experimental runs.

Given the choice, it would seem that the engineer would strongly prefer the maximally unconfounded design over the minimum aberration design. Having previously investigated the two-factor interaction effects of the original 4 factors, the preferable design would be the design that allows unconfounded estimates of the two-factor interactions involving the new factors under investigation.

The problem, however, is that the engineer likely would not have known that he or she had such a choice. To our knowledge, the originators of the minimal aberration criteria have not explicitly stated that minimizing the aberration of a design minimizes the number of pairs of aliased effects (a careful reading of the excerpt from Fries & Hunter, 1984, quoted near the beginning of this paper, reveals the potential ambiguity). The documentation for experimental design software also is often not explicit about the type of "confounding" that is minimized by minimum aberration designs (e.g., "Among all resolution IV designs, a design that allows you to estimate the maximum number of two factor-interactions is said to have *minimum aberration*." SAS Institute, Inc., 1995, p. 424, italics in original). This paper will have served its purpose if it helps to make the experimenter, contemplating the use of a fractional factorial design, aware of the possibility that there may be a better design available than the minimum aberration design.

This article also clarifies the characteristics of minimum aberration designs, and it is hoped that it will spur investigation of alternatives to minimum aberration as a criterion for the "best" design. Finally, it is hoped that experimental design software will be developed, with algorithms for finding maximally unconfounded designs as defined in this paper, to aid experimenters in the pursuit of the least ambiguous answers to their experimental questions, with a minimum of experimental effort.

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